Suppression of Higher-Order Terms of ADTS in Nonlinear Optimization of SPring-8 Upgrade

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- auxiliary weak sextupoles between arcs -

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- high-order formula by canonical perturbation and its use –

Modern map methods are useful, but we propose yet another tool:
it is "classical" but effective especially in the dsigning phase of lattice.

(4) Application to SPring-8-II Lattice Design

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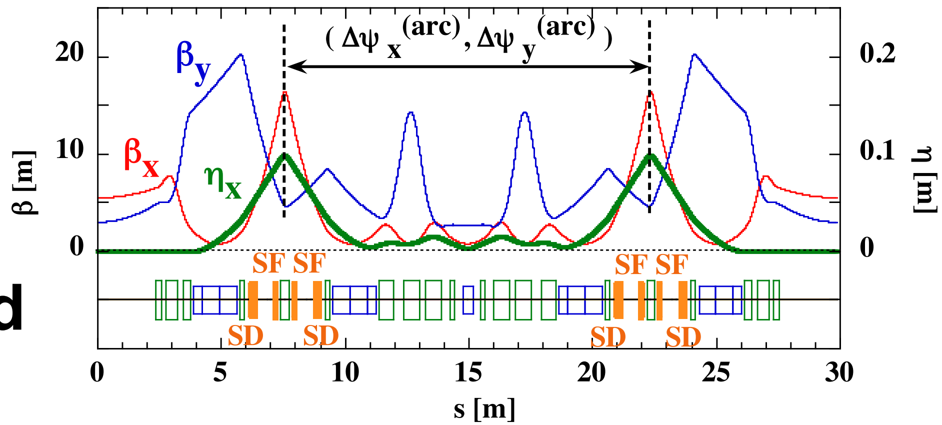
(1) Remarks on MBA Lattice Design

Multi-bend achromat (MBA) structure is generally used in designing a ring with very low emittance:

$$\varepsilon \propto (N_{BM} - 1)^{-3}$$ for simple MBA cell with $L_{BM}/2$ at both ends

As N_{BM} becomes larger, θ_{BM} is smaller and dispersion η_x in the arc takes smaller values, and then chromaticity correcting SXs become stronger.

The "hybrid MBA" lattice proposed by the ESRF has two dispersion bumps in a cell and low order contributions from SXs almost cancel due to betatron phase matching.



$\Delta\psi^{(arc)} \simeq (3\pi, \pi)$

This lattice has been adopted for the SPring-8 upgrade, however, ...

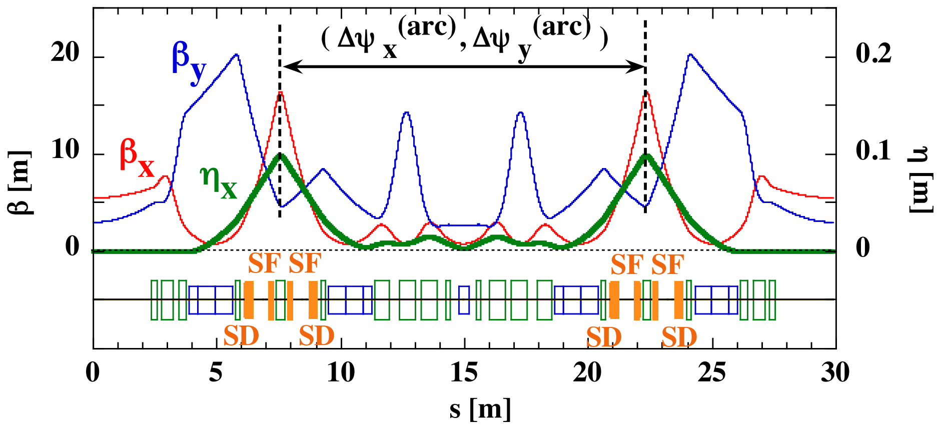
(1) Remarks on MBA Lattice Design

The number of independent knobs for nonlinear optimization is not many, and their tuning range is limited:

(1) Sextupoles (SF and SD) are localized in a narrow area around a peak of each dispersion bump, and so they almost degenerate in terms of betatron phase.

(2) Phase matching between arcs also impose a constraint, and phase relations among SXs are almost fixed.

(3) Tunable range of tune per cell ($\nu_x^{(cell)}$, $\nu_y^{(cell)}$) is limited since beta values at ID straights must be set to optimum values and the emittance is closely related to $\nu_x^{(cell)}$.



$\Delta\psi^{(arc)} \simeq (3\pi, \pi)$

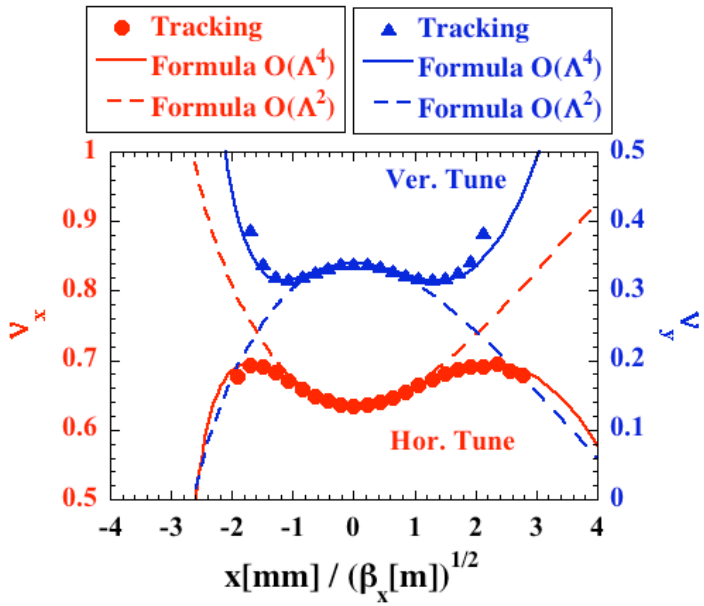
(1) Remarks on MBA Lattice Design

In addition, higher order terms of SXs dominate the dynamic stability and limit the dynamic aperture (DA).

A proper objective function in optimization is then difficult to set without taking acount of higher order terms.

So, the situation is very complicated and many people like to use genetic algorithms like MOGA ...

An example showing $O(\Lambda^4)$ terms dominate the amplitude-dep. tune shift (ADTS)



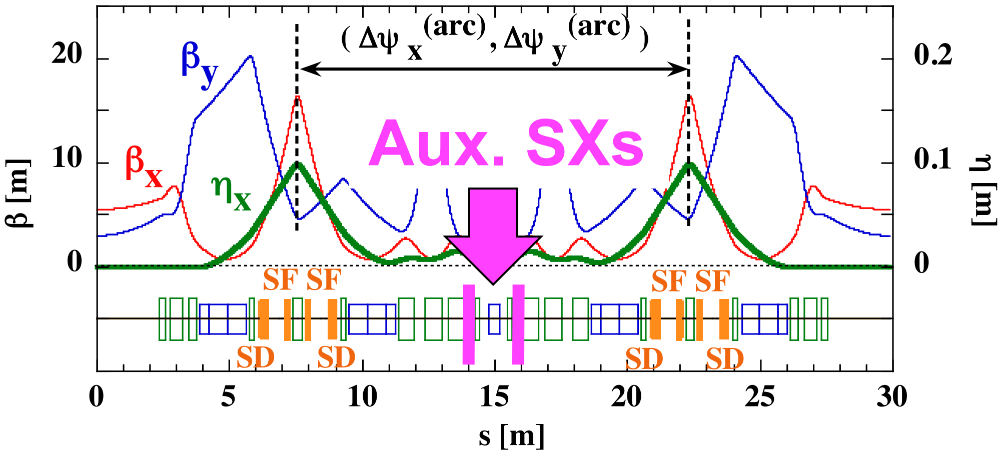
(2) A New Scheme for Suppressing Lattice Nonlinearity

Our Approach:

From a viewpoint of beam physics we should understand what limits the beam stability and think the way to solve the problem, not fully relying on genetic algorithms.

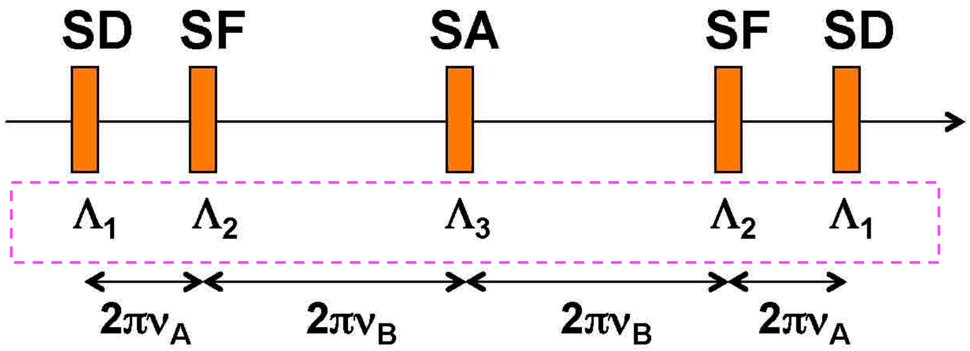
The origin of lattice nonlinearity such as high-order ADTS is the leakage (incomplete cancellation) of kicks by chromaticity correctiong SXs, and so the key point is how we can manage it.

To suppress the leakage kick, we propose to install additional weak SXs at around the middle of unit cell (H.Tanaka).



(2) A New Scheme for Suppressing Lattice Nonlinearity

Check with Toy Model:



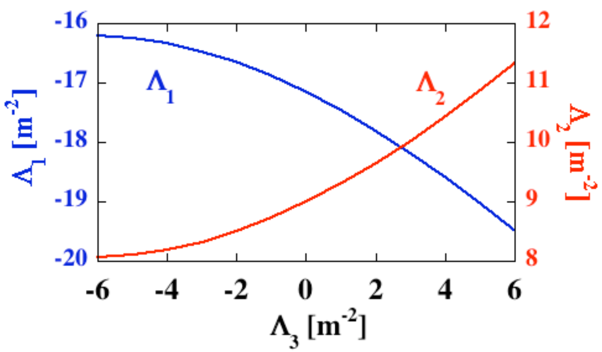
* One-dim. calc. (horizontal)
* SF and SD for chrom. cor.
* β_x = 5 m, α_x = 0 at all SX
  ν_A = 0.025, ν_B = 0.67

$$\Lambda_i \equiv \frac{-1}{2}\frac{B_i'' L_i}{[B\rho]}$$

SX strengths Λ_i are fixed as follows:

* **Lowest order ADTS coef. should vanish: $\partial\nu_x / \partial J_x = 0$**
* **Local chrom. should be kept with the assumption on dispersion $\eta_1 = \eta_2$: $\Lambda_1 + \Lambda_2$ = const.**

➔ **Λ_1 and Λ_2 are fixed as a function of Λ_3.**



(2) A New Scheme for Suppressing Lattice Nonlinearity

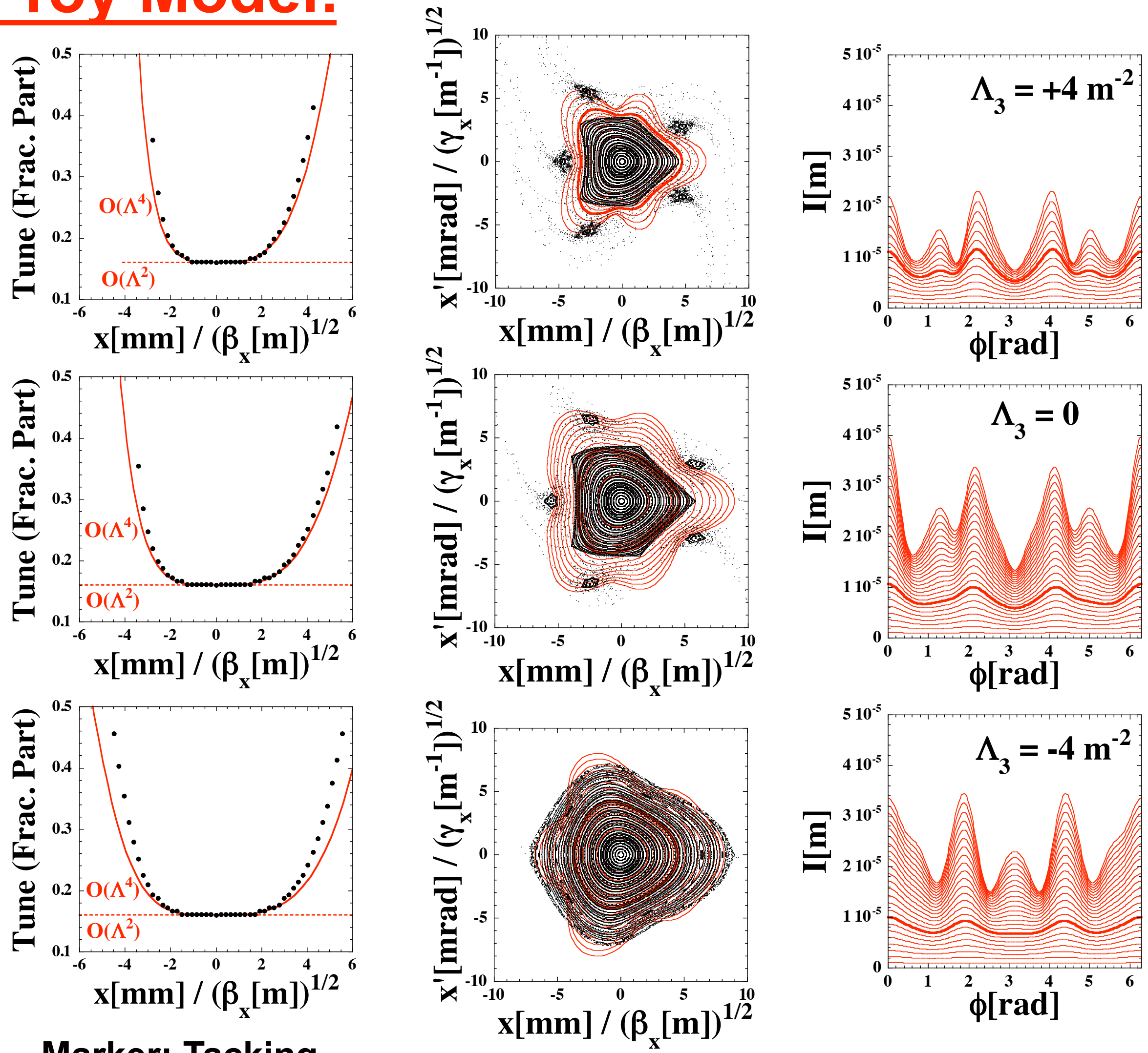
Check with Toy Model:

ADTS, Phase Space, and Action-Angle

Λ_3 = +4

Λ_3 = +0

Λ_3 = -4



* Bold curve:
J = 8e-6 [m]
(for guiding eyes)

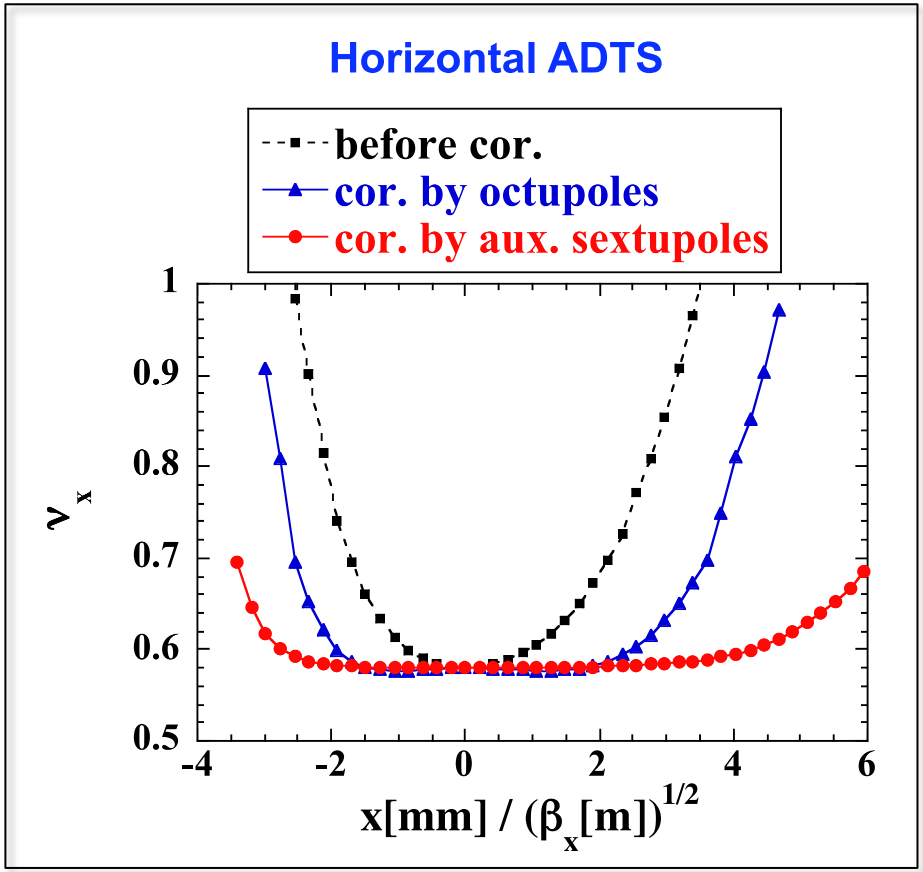
Marker: Tacking

Solid Curves: Canonical Pert. Cal. up to O(Λ^4)

(2) A New Scheme for Suppressing Lattice Nonlinearity

Application to SPring-8-II Lattice:

Necessary strength is about one order of magnitude smaller than chromaticity correcting SXs, and the range of flatness is wider than the case using octupoles, which means that higher order terms are well controlled.



For details, see:

K.Soutome et al., Proc. IPAC17, p.3091.

(3) Formula of Higher-Order ADTS

In the MBA lattice with strong SXs, the ADTS is dominated by the terms of O(Λ^4) (or higher). To control such high-order terms, it is necessary to know their response to SX.

→ **We did canonical perturbation calculations up to O(Λ^4) by neglecting terms of O(J_y^2).**

This is adequate since large DA is required mainly in the horizontal direction, and the Hamiltonian is written as

$$\tilde{H} = \frac{J_x}{\beta_x(s)} + \frac{J_y}{\beta_y(s)} + W_{xx}(s)J_x^2 + W_{xy}(s)J_xJ_y + W_{xxx}(s)J_x^3 + W_{xxy}(s)J_x^2J_y + \cdots$$

$$\Rightarrow \quad \tilde{\nu}_x = \frac{1}{2\pi}\int ds \frac{\partial\langle\tilde{H}\rangle}{\partial J_x} = \nu_x + 2c_{xx}J_x + c_{xy}J_y + 3c_{xxx}J_x^2 + 2c_{xxy}J_xJ_y$$

$$\tilde{\nu}_y = \frac{1}{2\pi}\int ds \frac{\partial\langle\tilde{H}\rangle}{\partial J_y} = \nu_y + c_{xy}J_x + 2c_{yy}J_y + c_{xxy}J_x^2$$

O(Λ^2) O(Λ^4) : ***new formula***

For canonical perturbation theory, see e.g. R.D.Ruth, AIP Conf.Proc.153(1987)150.

(3) Formula of Higher-Order ADTS

SX-separated form of coefficients:

$$c_{\alpha\beta} = \sum_{i,j} \lambda_i \lambda_j F_{\alpha\beta}^{(ij)}$$

$$c_{xxx} = \sum_{i,j,k,l} \lambda_i \lambda_j \lambda_k \lambda_l F_{xxx}^{(ijkl)}$$

$$c_{xxy} = \sum_{i,j,k,l} \lambda_i \lambda_j \lambda_k \lambda_l F_{xxy}^{(ijkl)}$$

$$\lambda_i \equiv \frac{B_i''}{[B\rho]} \left(= -\frac{2}{L_i} \Lambda_i \right)$$: strength of i-th SX

Coefficients $F_{\alpha\beta}^{(ij)}$ and $F_{\alpha\beta\gamma}^{(ijk)}$ are calculated by the Twiss parameters, and their explicit expressions are given in K.Soutome and H.Tanaka, PR-AB 20 (2017) 06401.

$$F_{\alpha\beta}^{(ij)} \equiv \frac{1}{2\pi}\int_0^C ds \tilde{W}_{\alpha\beta}^{(ij)}(s)$$

$$F_{\alpha\beta\gamma}^{(ijkl)} \equiv \frac{1}{2\pi}\int_0^C ds \tilde{W}_{\alpha\beta\gamma}^{(ijkl)}(s)$$

where, for example,

$$\begin{aligned}
\tilde{W}_{xxx}^{(ijkl)}(s) \equiv\ & 2\mathrm{Re}\tilde{A}_1^{(i)}(s)\Big[\frac{1}{24}\Big(\tilde{B}_1^{(j)}\tilde{B}_1^{(k)}\tilde{B}_1^{(l)} + 9\tilde{B}_1^{(j)}\tilde{B}_1^{(k)}\tilde{B}_1^{(l)*} \\
& +6\tilde{B}_1^{(j)}\tilde{B}_1^{(k)*}\tilde{B}_2^{(l)} + 9\tilde{B}_1^{(j)}\tilde{B}_1^{(k)}\tilde{B}_2^{(l)*} + 18\tilde{B}_1^{(j)}\tilde{B}_2^{(k)}\tilde{B}_2^{(l)*} + 3\tilde{B}_2^{(j)}\tilde{B}_2^{(k)}\tilde{B}_2^{(l)*}\Big) \\
& +\Big(\tilde{B}_1^{(j)} + 3\tilde{B}_1^{(j)*} + 3\tilde{B}_2^{(j)*}\Big)\tilde{C}_1^{(kl)} + 2\Big(\tilde{B}_1^{(j)*} + 3\tilde{B}_2^{(j)*}\Big)\tilde{C}_2^{(kl)} + 3\tilde{B}_2^{(j)*}\tilde{C}_3^{(kl)} \\
& -3\tilde{D}_1^{(jkl)} - 3\tilde{D}_2^{(jkl)}\Big]_s
\end{aligned}$$

with

$$\tilde{A}_1^{(i)}(s) \equiv \frac{1}{8\sqrt{2}}\Delta_i(s)\beta_x^{3/2}(s) \qquad \text{etc.}$$

$$\Delta_i(s) = \begin{cases} 1, & s_i \le s \le s_i + L_i \\ 0, & otherwise \end{cases}$$

$$\tilde{B}_1^{(i)}(s) \equiv \frac{1}{\sin\pi\nu_x}\int_s^{s+C} ds'\tilde{A}_1^{(i)}(s')e^{i\Psi_x(s',s)} \qquad \text{etc.}$$

$$\Psi_x(s',s) \equiv \psi_x(s') - \psi_x(s) - \pi\nu_x$$

i, j, k: index of sextupoles

$$\tilde{C}_1^{(ij)}(s) \equiv \frac{1}{2\sin 2\pi\nu_x}\int_s^{s+C} ds'\Big[-\tilde{A}_1^{(i)}\Big(3\tilde{B}_1^{(j)} + \tilde{B}_1^{(j)*} + 3\tilde{B}_2^{(j)}\Big)\Big]_{s'} e^{i2\Psi_x(s',s)} \qquad \text{etc.}$$

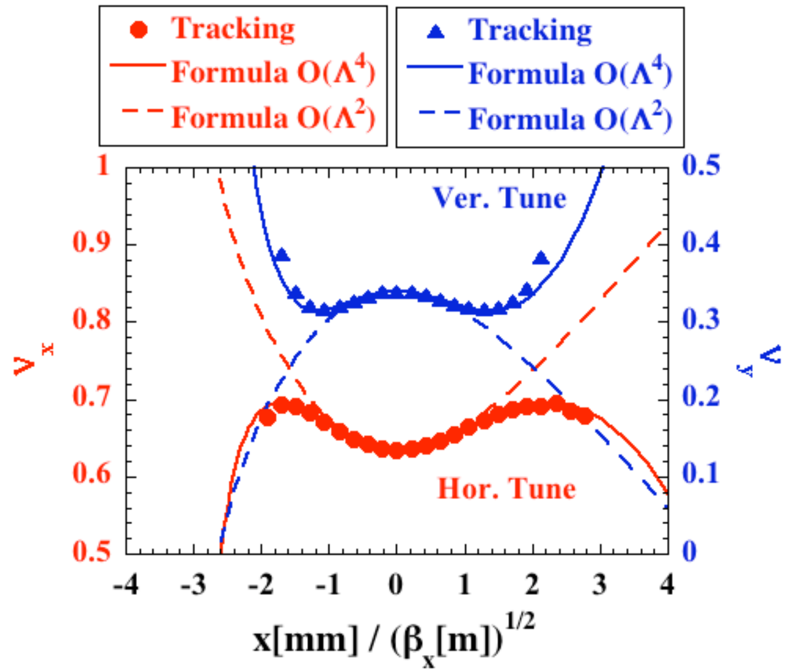
$$\begin{aligned}
\tilde{D}_1^{(ijk)}(s) \equiv \frac{1}{2\sin\pi\nu_x}\int_s^{s+C} ds'\Big[\tilde{A}_1^{(i)}\Big\{\frac{1}{4}\Big(& 3\tilde{B}_1^{(j)}\tilde{B}_1^{(k)} + 6\tilde{B}_1^{(j)}\tilde{B}_1^{(k)*} + \tilde{B}_1^{(j)*}\tilde{B}_1^{(k)*} + 2\tilde{B}_1^{(j)}\tilde{B}_2^{(k)} + 6\tilde{B}_1^{(j)*}\tilde{B}_2^{(k)} + 2\tilde{B}_1^{(j)}\tilde{B}_2^{(k)*} + 6\tilde{B}_2^{(j)}\tilde{B}_2^{(k)*}\Big) \\
& -6\tilde{C}_1^{(jk)} - 2\tilde{C}_1^{(jk)*} - 4\tilde{C}_2^{(jk)}\Big\}\Big]_{s'} e^{i\Psi_x(s',s)} \qquad \text{etc.}
\end{aligned}$$

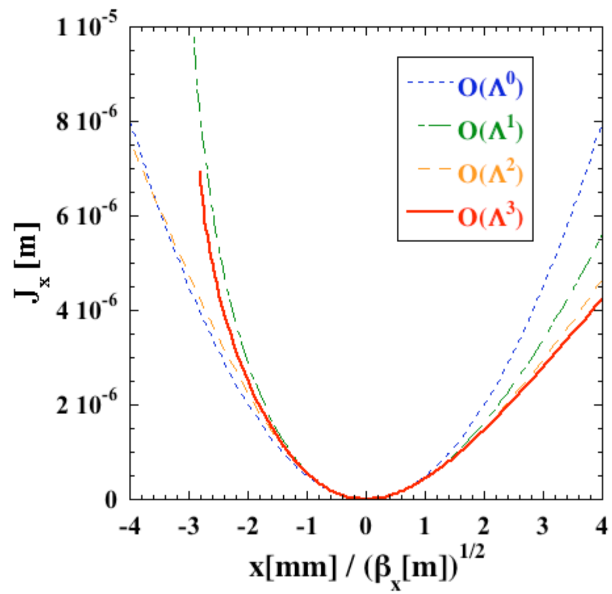
By using complex variables, direct coding with C is possible.
Note: For the ring with N cells, ADTS coef. is N times that of one unit cell.

(3) Formula of Higher-Order ADTS

The higher-order formula can be incorporated in the objective function of SX optimization procedure, and the horizontal ADTS was evaluated up to O(Λ^4).

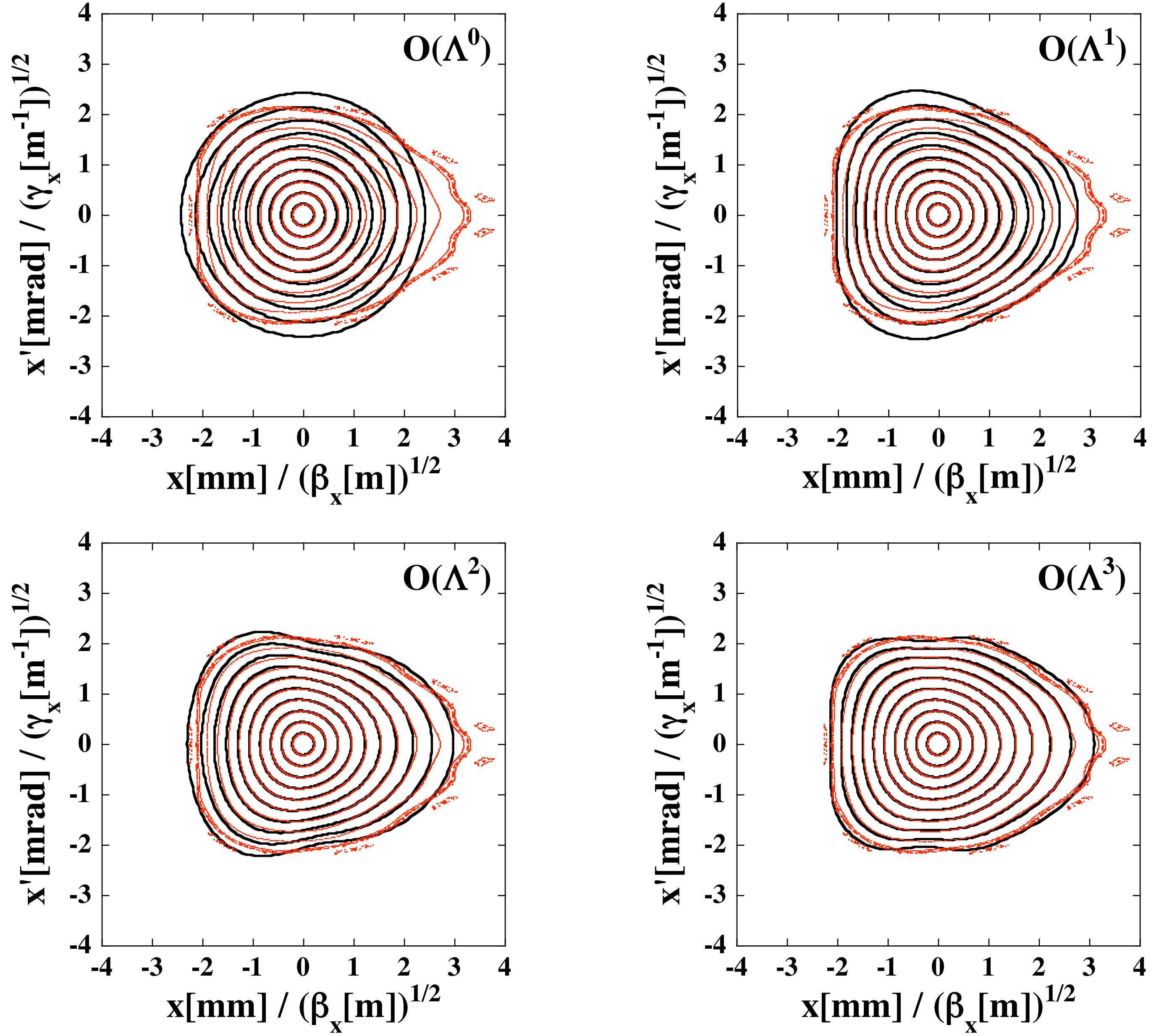
An example case calculated with the formula up to O(Λ^4)





The relation between the action J_x and coordinate x is distorted at large amplitudes, and a naive lowest order treatment is inadequate.

(3) Formula of Higher-Order ADTS



Marker: Tacking

Solid Curves:
Canonical Pert.
Cal. up to $O(\Lambda^4)$

(4) SPring-8-II Lattice Design

- Optimization of Unit Cell -

Tuning Knobs in Nonlinear Optimization:

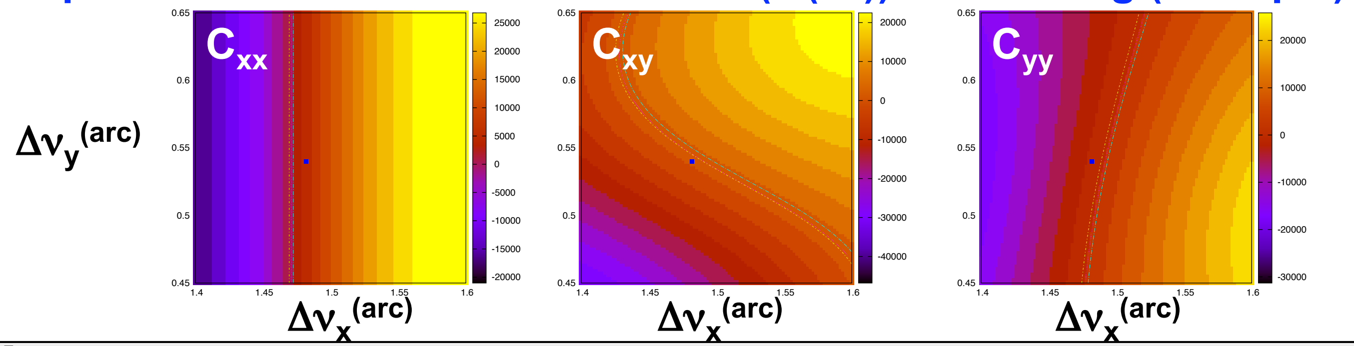
* SX excitation pattern under the constraint of fixed chrom.

  5 families in a cell: { SD_1 SF_1 SF_2 SD_2 S_{aux} | S_{aux} SD_2 SF_2 SF_1 SD_1 }

* detuning of phase between arcs: ($\Delta\nu_x^{(arc)}$, $\Delta\nu_y^{(arc)}$)

* tune/cell

Response of lowest-order ADTS coef. ($O(\Lambda^2)$) to detuning (example)

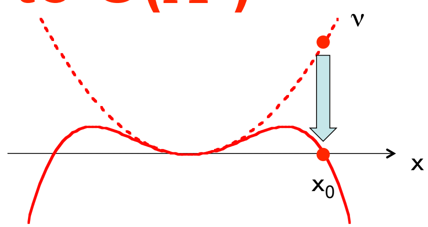


Objective Function:

* ADTS with the use of perturbation formula up to $O(\Lambda^4)$

* 1st and 2nd order terms of chromaticity

* resonance driving terms (when needed)



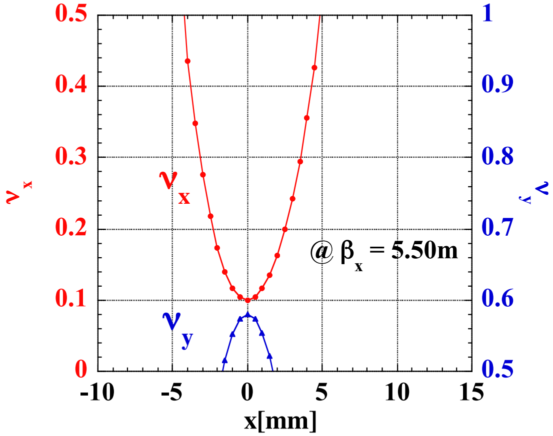
NB: We also have octupoles but these are saved as a future knob.

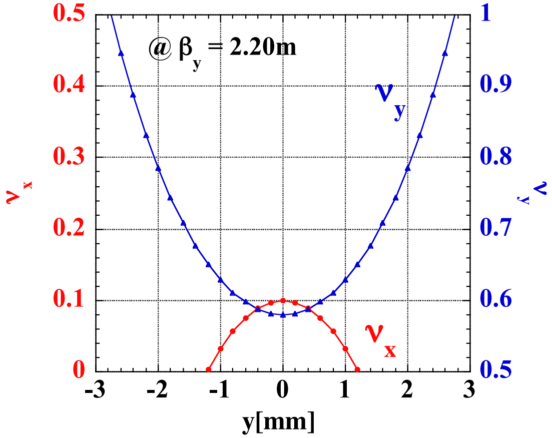
(4) SPring-8-II Lattice Design

- Optimization of Unit Cell -

ADTS

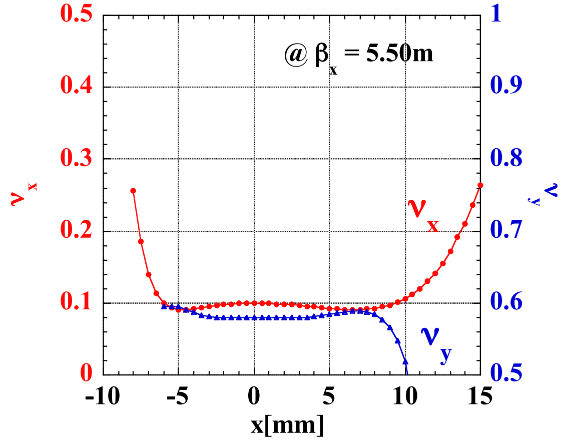
before optimizing SXs

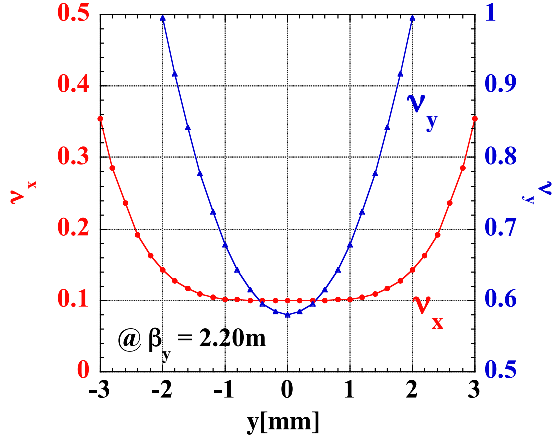


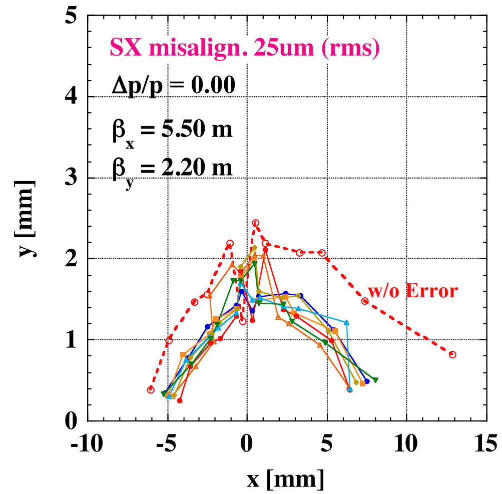


ADTS and DA

after optimizing SXs







DA @ β_x = 5.5m

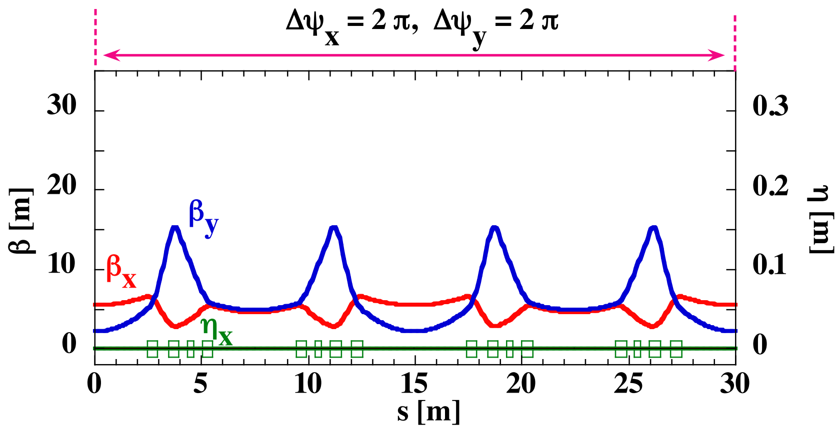
(4) SPring-8-II Lattice Design

- Insertion of LSS and Injection Section -

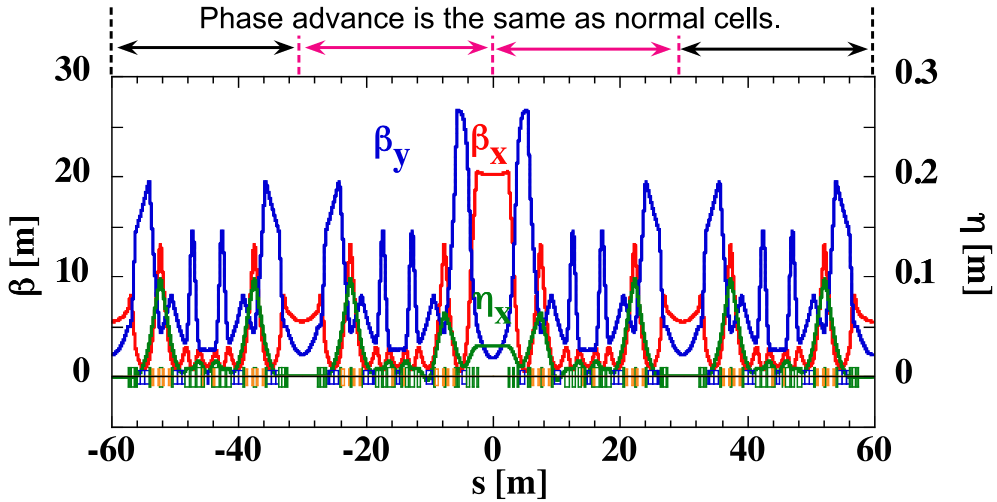
The ring has four long straight sections (LSS) of about 30m and an injection section having high horizontal beta of β_x = 20m.

In both sections phase matching condition is imposed and these sections are transparent for on-momentum electrons.

LSS

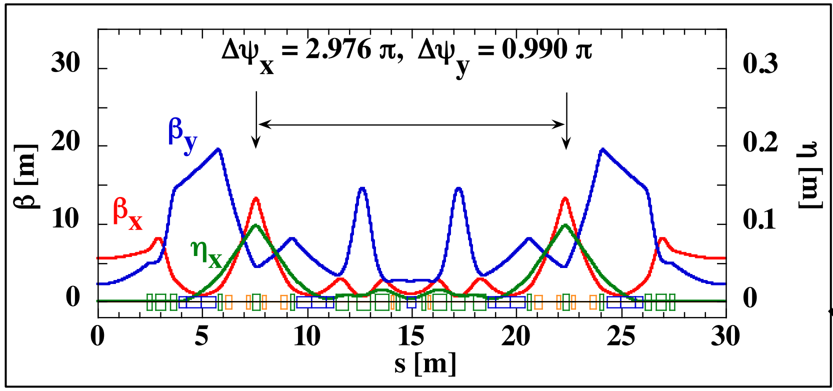


Injection Section

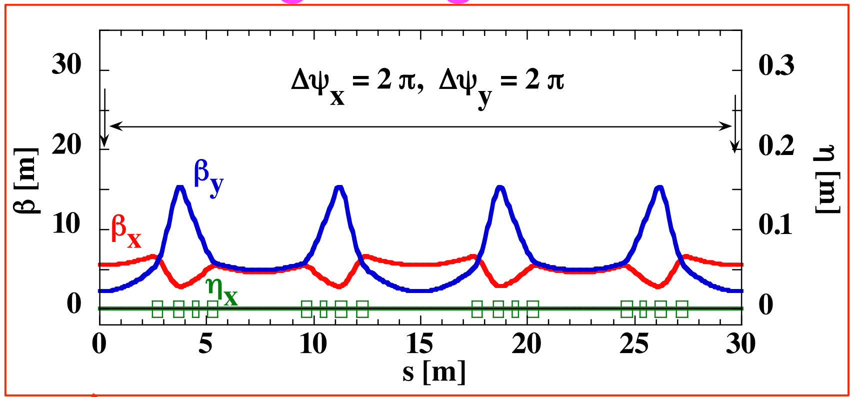


Ring Structure

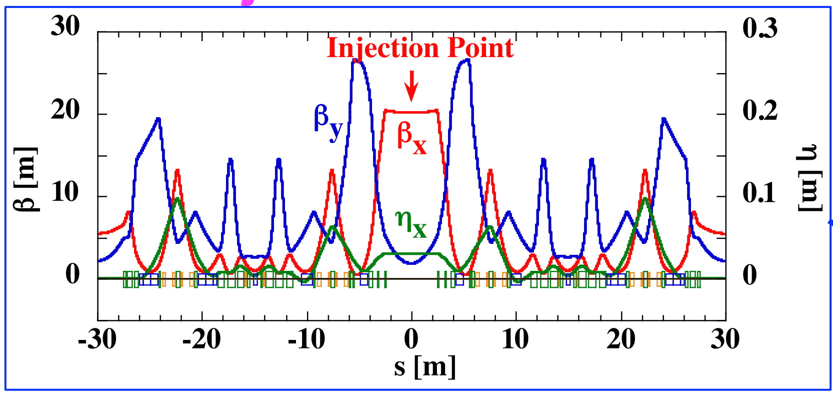
Unit Cell

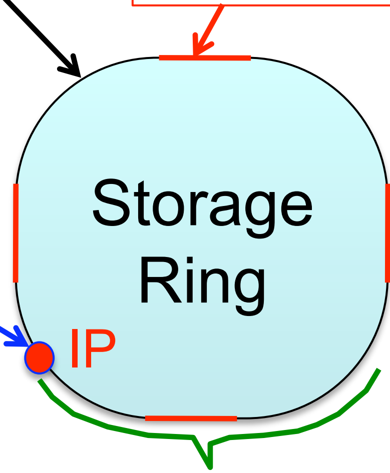


Long Straight Cell

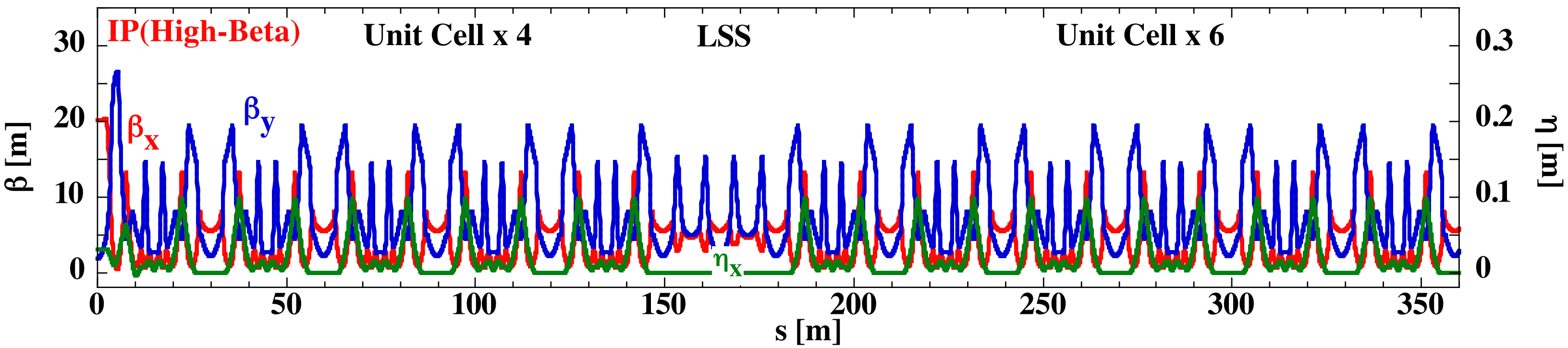


Injection Cells



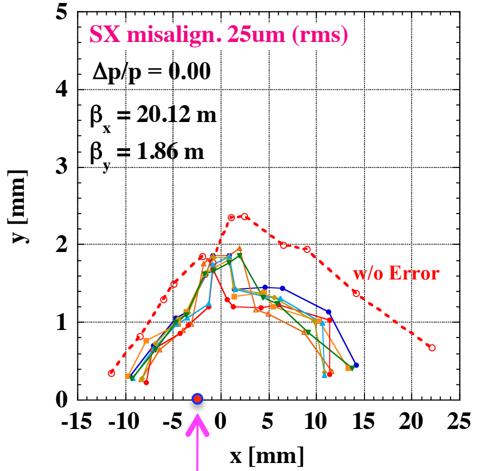


42 Unit Cells
+ 2 Injection Cells
+ 4 Long Straight Cells



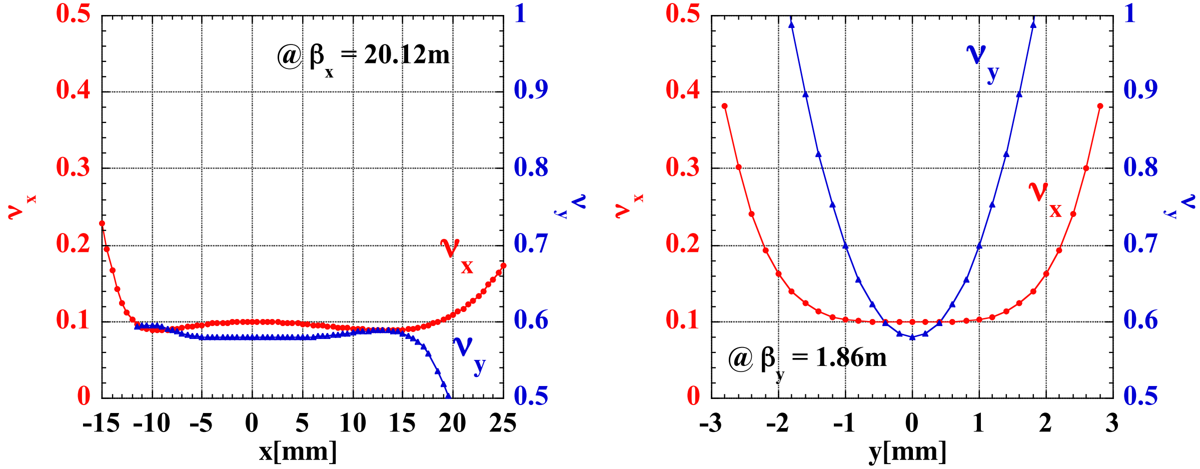
Dynamics

Dynamic Aperture (@ IP)

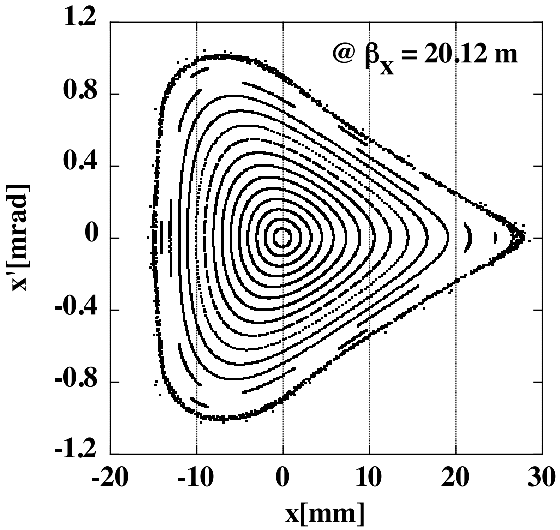


injected beam (@ x = -2〜-3mm) from SACLA

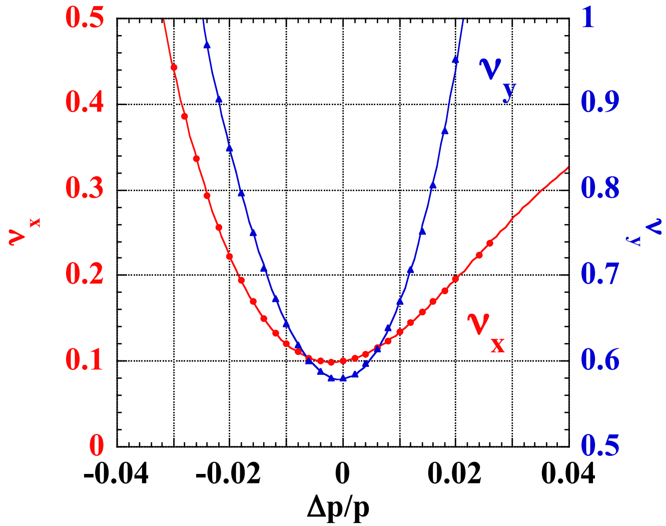
Amplitude-Dep. Tune Shift



Hor. Phase Space (@ y=0)



Chromaticity



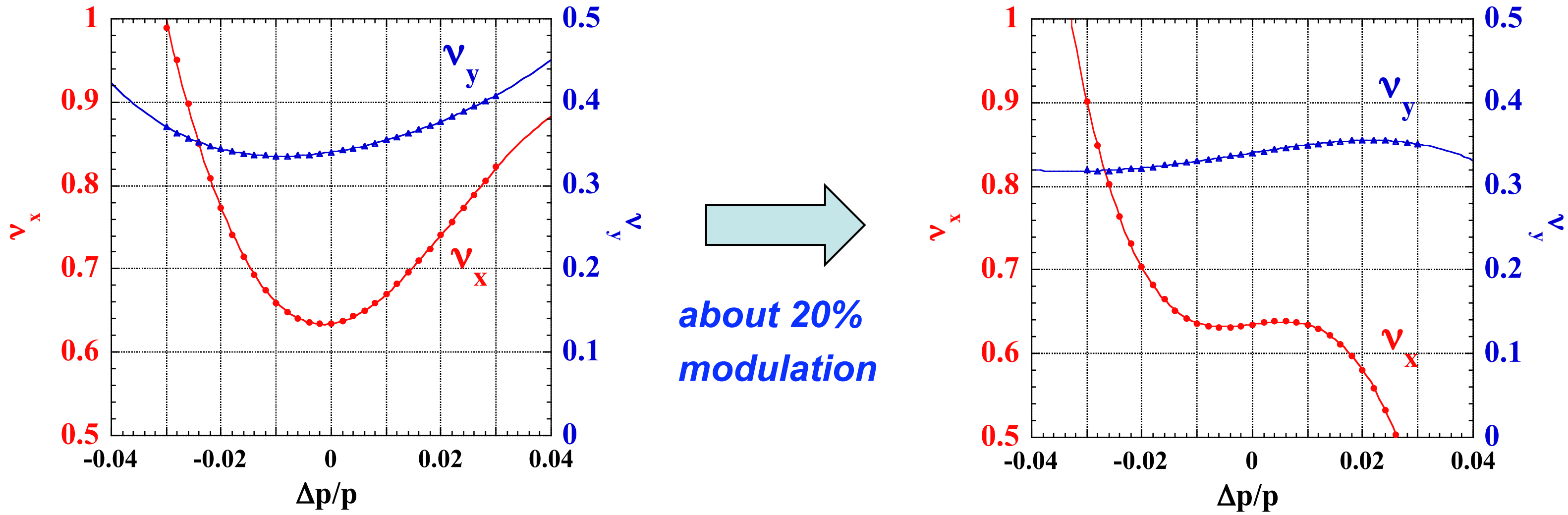
Machine Parameters (as of Sep. 2017)

	SPring-8-II	SPring-8 (Present)
E [GeV]	6	8
I [mA]	100	100
C [m]	1435.45	1435.95
Lattice	5BA (w/ Long. Var.)	DB
ε [nmrad]	0.157 ~0.10 w/ undulators	6.6 (Achromat) , 2.4 (Non-Achromat)
(β_x, β_y) [m] @ ID	(5.5, 2.2)	(24.4, 5.8) (A) , (31.2, 5.0) (NA)
η_x [m] @ ID	0.0	0.0 (A) , 0.146 (NA)
(ν_x, ν_y)	(108.10, 44.58)	(40.15, 18.35) (A) , (41.14, 19.35) (NA)
$(\xi_x, \xi_y)_{natural}$	(-143, -147)	(-90, -41) (A) , (-117, -47) (NA)
α	3.24e-5	1.46e-4 (A) , 1.60e-4 (NA)
$\sigma_{\Delta p/p}$ [%]	0.093	0.109
κ [%]	10	0.2
h	2436	2436
f_{RF} [Hz]	508.76	508.58
ΔU [MeV/turn]	2.96	9.12

Nonlinear Chromaticity

To obtain enough Touschek beam lifetime, the momentum acceptance (MA) must be as large as possible. The MA of the present lattice is about 2% and we are trying to suppress the 2nd order chromaticity $\xi^{(2)}_{x,y}$ by extending the SX excitation pattern to plural cells.

An example of $\xi^{(2)}$-control with 3-cell modulation of SX



Studies are ongoing to find an optimum set of SXs to suppress $\xi^{(2)}_{x,y}$ and ADTS at the same time.

(5) Summary and Future Outlook

* We proposed a new scheme of introducing weak SXs for suppressing leakage kick due to chrom. correcting SXs.
=> effective for suppressing ADTS and obtaining a large DA

* We also developed a higher-order formula of ADTS up to $O(\Lambda^4)$ and the formula can be used to set a proper objective function.

* A commissioning scenario (First-Turn Steering, beam accumulation, COD and optics cor., ...) has been studied, and the final beam peformance is expected to be good enough.

* Nonlinear chromaticity still needs to be optimized to get a larger mom. acceptance.

* Lattice optimization is still ongoing aiming at better performance (working point, sextupoles and octupoles ...).